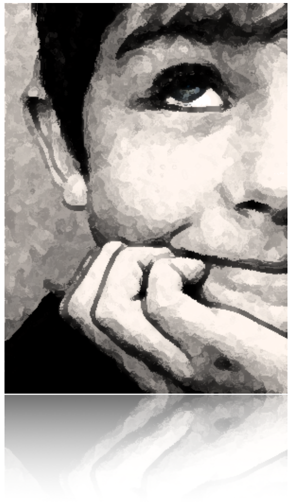
Prologue, last week

RV
ST ensemble
CPDF
Prior
Likelihood
Eigenvector Decomposition
Covariance



$$r_{\mathrm{est}}(t) = r_0 + \int_0^\infty d\tau\, D(\tau)s(t-\tau)$$

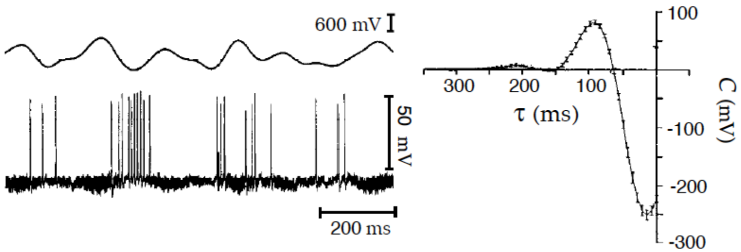
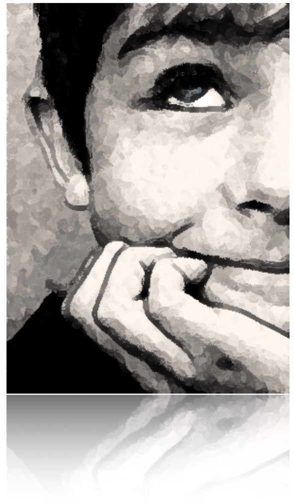


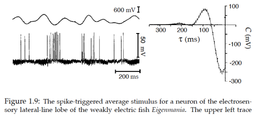
Figure 1.9: The spike-triggered average stimulus for a neuron of the electrosensory lateral-line lobe of the weakly electric fish *Eigenmannia*. The upper left trace

Encoding & decoding with probabilistic brain

Prologue, today

Linear kernel
RV
Nonlinearity
Poisson
PCA
Eigenvector Decomposition
Covariance

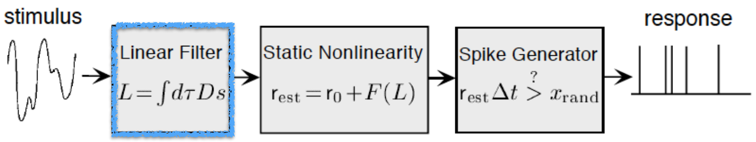




$$r_{\mathrm{est}}(t) = r_0 + \int_0^\infty d\tau\, D(\tau)s(t-\tau)$$

Volterra expansion expansion. For the case we are considering, it takes the form

$$r_{\mathrm{est}}(t) = \boxed{r_0 + \int d\tau\, D(\tau)s(t-\tau)} + \int d\tau_1 d\tau_2\, D_2(\tau_1,\tau_2)s(t-\tau_1)s(t-\tau_2) + \int d\tau_1 d\tau_2 d\tau_3\, D_3(\tau_1,\tau_2,\tau_3)s(t-\tau_1)s(t-\tau_2)s(t-\tau_3) + \ldots \quad (2.2)$$



$$r_{est}(t) = r_0 + \boxed{\int_0^{\infty} d\tau\, D(\tau) s(t-\tau)}$$

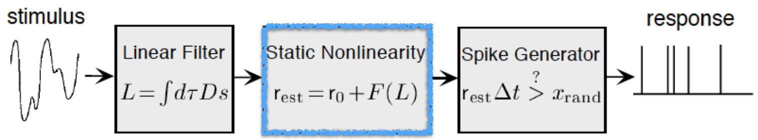
$$E = \frac{1}{T}\int_0^T dt\, (r_{est}(t) - r(t))^2\,. \qquad (2.3)$$

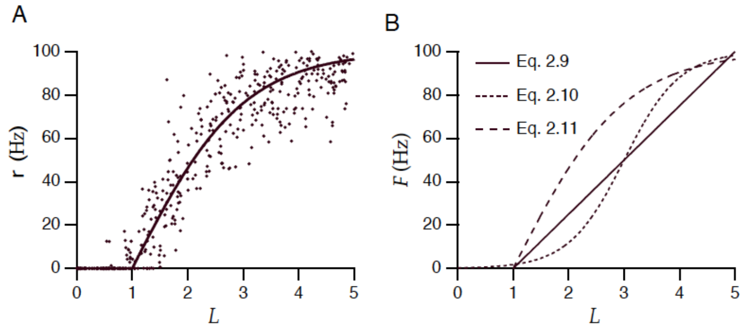
optimal $D(\tau)$ = $\underset{D(\tau)}{\text{arg min}}$ (E)

if s(t) sampled from white noise

$Q_{rs}(\tau) = \int dt\, r(t)\, s(t+\tau)/T,$

optimal $D(\tau) = \dfrac{Q_{rs}(-\tau)}{\sigma_s^2} = \dfrac{\langle r\rangle C(\tau)}{\sigma_s^2}$ — STA

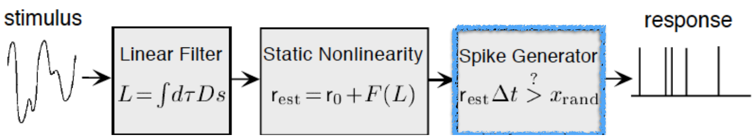




$$F(L) = G[L - L_0]_+ \qquad (2.9)$$

$$F(L) = \frac{r_{max}}{1 + \exp\left(g_1(L_{1/2} - L)\right)}\,. \qquad (2.10)$$

$$F(L) = r_{max}[\tanh\left(g_2(L - L_0)\right)]_+ \qquad (2.11)$$



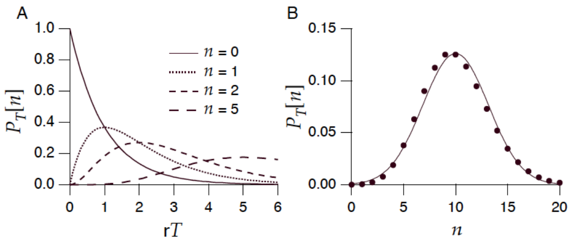
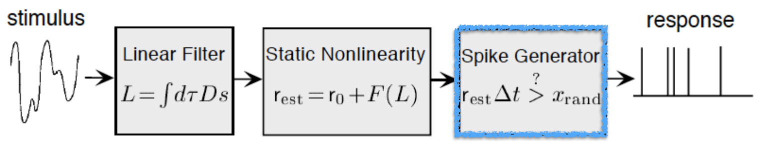


Figure 1.11: A) The probability that a homogeneous Poisson process generates n spikes in a time period of duration T plotted for $n = 0$, 1, 2, and 5. The probability

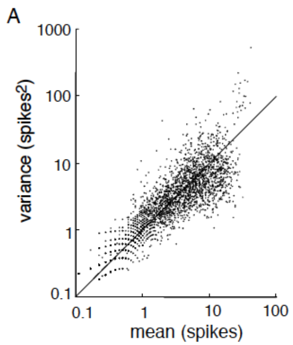
$$P_T[n] = \frac{(rT)^n}{n!}\exp(-rT) \longrightarrow \sigma_n^2 = \langle n^2\rangle - \langle n\rangle^2 = rT$$

Fano factor

Thus the variance and mean of the spike count are equal. The ratio of these two quantities, $\sigma_n^2/\langle n\rangle$, is called the Fano factor and takes the value one for a homogeneous Poisson process, independent of the time interval T.



$$P_T[n] = \frac{(rT)^n}{n!}\exp(-rT) \longrightarrow \sigma_n^2 = \langle n^2\rangle - \langle n\rangle^2 = rT$$



Variability of MT neurons in alert macaque monkeys responding to moving visual images. Variance of the spike counts for a 256 ms counting period plotted against the mean spike count. The straight line is the prediction of the Poisson model. Data are from 94 cells recorded under a variety of stimulus conditions. (Adapted from O'Keefe et al., 1997.)

RV and Neural computation

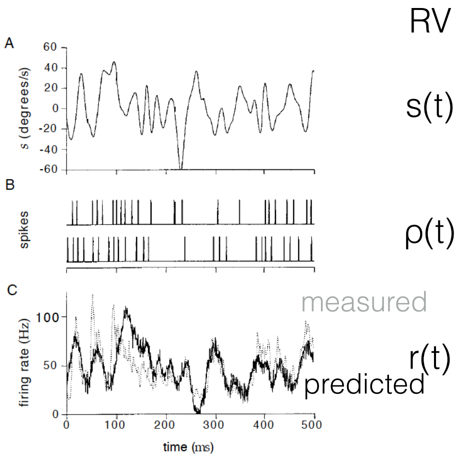
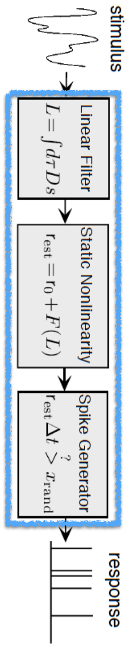
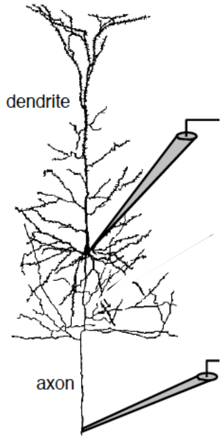
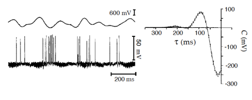


Figure 2.1: Prediction of the firing rate for an H1 neuron responding to a moving visual image. A) The velocity of the image used to stimulate the neuron. B) Two of the 100 spike sequences used in this experiment. C) Comparison of the measured and computed firing rates. The dashed line is the firing rate extracted directly from the spike trains. The solid line is an estimate of the firing rate constructed by linearly filtering the stimulus with an optimal kernel. (Adapted from Rieke et al., 1997.)







$$r_{est}(t) = r_0 + \int_0^\infty d\tau\, D(\tau)s(t-\tau)$$

Volterra expansion expansion. For the case we are considering, it takes the form

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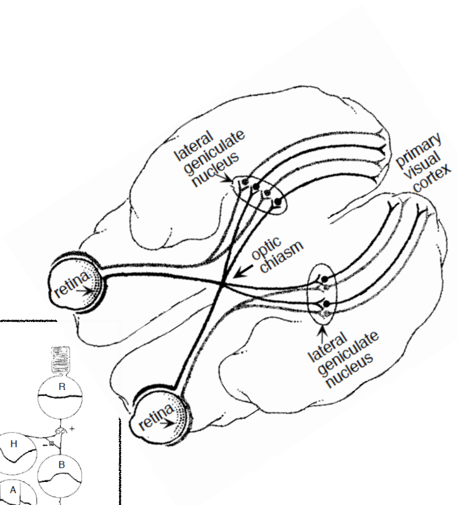
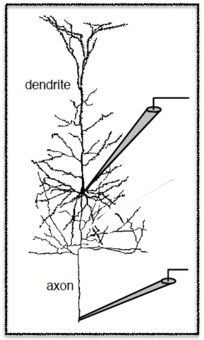
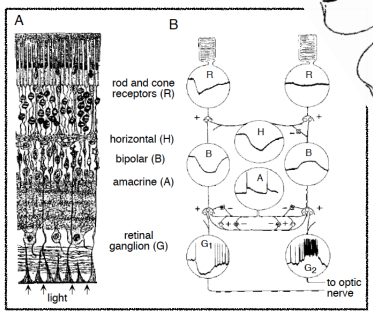
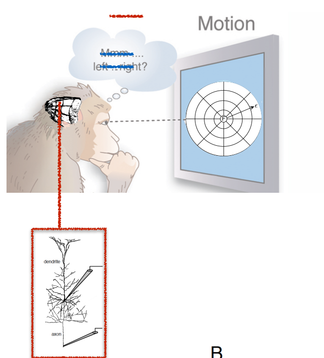
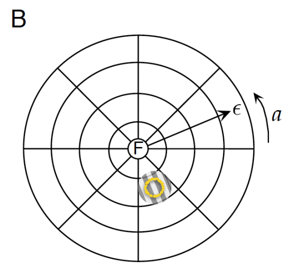
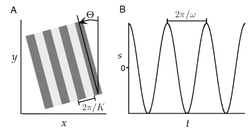


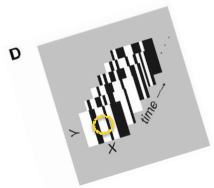


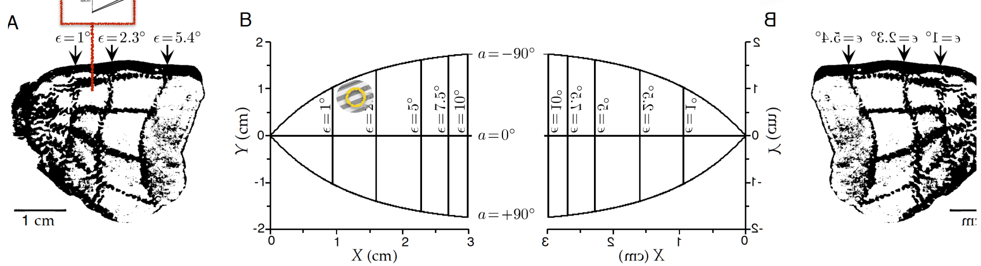












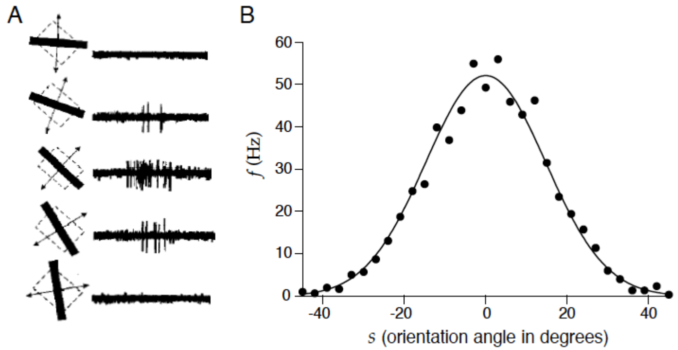


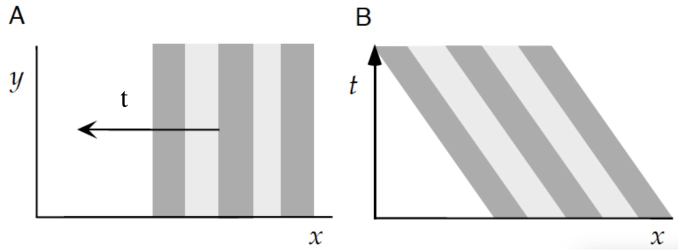
Figure 1.5: A) Recordings from a neuron in the primary visual cortex of a monkey.

Gaussian tuning curve

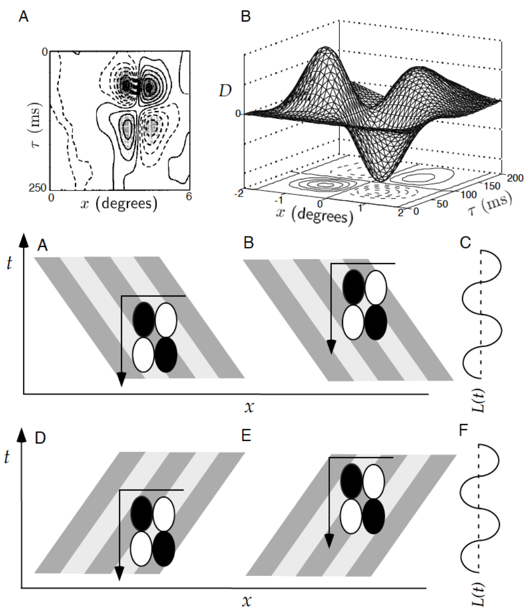
$$f(s) = r_{\max} \exp\left(-\frac{1}{2}\left(\frac{s - s_{\max}}{\sigma_f}\right)^2\right) \qquad (1.14)$$

A Separable vs Non-separable space-time receptive field

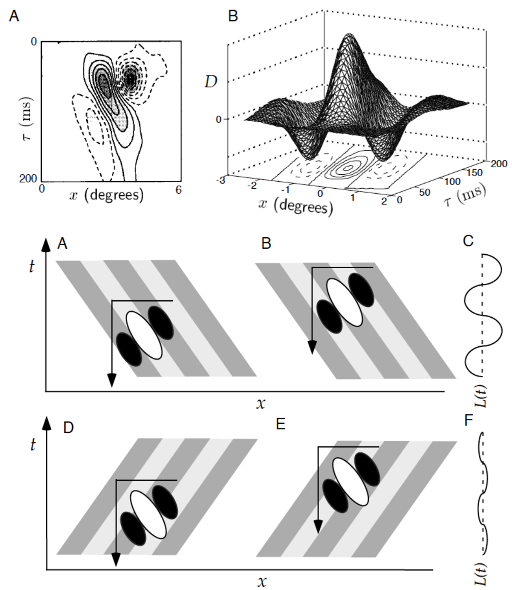
from a Space diagram to a Space-Time diagram of a moving grating

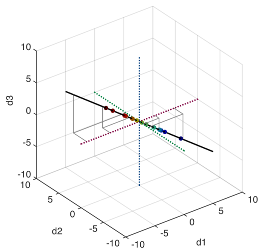


A Separable space-time receptive field



A Non-separable space-time receptive field





SE =

d1	d2	d3
-5.6745	-1.8915	-3.7830
3.4782	-1.1594	-2.3188
3.0071	-1.0024	-2.0048
2.8249	-0.9416	-1.8833
2.7309	-0.9103	-1.8206
2.0255	-0.6752	-1.3504
1.9635	-0.6545	-1.3090
0.9409	-0.3136	-0.6273
0.3395	-0.1132	-0.2264
-0.3548	0.1183	0.2366
-0.9205	0.3068	0.6136
-0.9770	0.3257	0.6513
-1.8585	0.6195	1.2390
-2.0261	0.6754	1.3508
-3.6291	1.2097	2.4194
-4.5677	1.5226	3.0451

Principal Composition Analysis of ***data*** Matrix, SE

Eigenvalue Decomposition of ***covariance*** Matrix, ***C***

"calculating the eigenvectors and eigenvalues"

$$Ce = \lambda e$$
$$(C - \lambda I)e = 0$$
$$\det(C - \lambda I) = 0$$

$$E\ diagonalizes\ C$$
$$E^{-1}CE = D$$
$$\text{linearly independent } e_k$$

"find ***E*** who can diagonalizes ***C***"

C

	d1	d2	d3
d1	7.77	-2.59	-5.18
d2	-2.59	0.86	1.72
d3	-5.18	1.72	3.45

CoVariance

Variance

D=

d11	0	0
0	d22	0
0	0	d33

Matlab Demo1
visual illustration of PCA

exercise: Eigenvector Decomposition of 2 x 2 matrix

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$

find λs and es satisfying

$$Ce = \lambda e$$
$$(C - \lambda I)e = 0$$
$$\det(C - \lambda I) = 0$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

$$\begin{vmatrix} 3-\lambda & 2 \\ 3 & -2-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - \lambda - 12 = 0$$

$$for\ e_1\ belonging\ to\ \lambda_1 = 4,$$
$$A - 4I = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$$

$$[A - 4I]e = 0$$

$$e_1 = (2x, x)^T$$

$$for\ e_2\ belonging\ to\ \lambda_2 = -3,$$
$$A + 3I = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$$

$$[A + 3I]e = 0$$

$$e_2 = (-x, 3x)^T$$

$$E = x[e_1, e_2] = x\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Matlab Demo 2
PCA of Data that were generated by
forward-engineering of Rust et al. (2005)

Homework for next week

1. Writing matlab codes to recover linear filters
2. Writing Methods/Results section of 1.